# ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

## ACCURACY OF THE GRADIENT METHOD IN HEAT TRANSFER

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UDC 536.24.083

The gradient method is an experimental method of determining the local heat-transfer coefficient at the surface; the fundamental working formula is based on Fourier's law and takes the form

$$\alpha = -\lambda \frac{\left(\frac{\partial t}{\partial n}\right)_{n=0}}{t_f - t_w}. \tag{1}$$

The temperature gradient at the transfer surface $(\partial t/\partial n)_{n=0}$ is found by determining the temperature distribution at the wall subject to boundary conditions of the first kind (Dirichlet problem), which themselves are defined by experiment. If there are heat losses and longitudinal heat leaks, these can be incorporated in the solution to the differential equation along with the boundary conditions, which are specified at the edge of a longitudinal section of the body (axially symmetrical case).

If the shape is complicated (nozzle, tip of a rocket, turbine blade, etc.) or if the boundary conditions are complicated, then the temperature distribution may be determined numerically; the principles of the gradient method for such conditions have previously been given [1], together with experimental tests.

The paper presents an evaluation of the relative error in the determination of heat-transfer coefficients in this way for a cylindrical surface under steady-state conditions, together with a numerical solution for the temperature distribution. The maximum value for the relative error is defined by

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta\lambda}{\lambda} + \frac{\Delta\left[\left(\frac{\partial t}{\partial r}\right)_{r=R_1}\right]}{\left(\frac{\partial t}{\partial r}\right)_{r=R_1}} + \frac{\Delta t_f + \Delta t_w}{t_f - t_w}, \tag{2}$$

which is used in estimating the individual components in (2); appropriate allowance has been made for the error in numerical solutions of the problem (by Runge's method), particularly from the replacement of $(\partial t/\partial r)_{r=R_1}$ by the ratio of finite differences (computer calculation), the errors in recording the boundary temperatures, errors due to effects of slots under thermocouples (in that case by computer calculation and electrothermal analogy techniques), and so on. A graph is given defining the relative error in surface temperature determination for various sizes of slot and various Biot numbers at the surface.

In one of the experiments ($Re_f = 9.1 \cdot 10^4$, $Bi_1 = 0.244$, $t_{f_1} = 344$°C), an estimate was made of the relative error in the local heat-transfer coefficient; it was found that this error was 19% for those working conditions.

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# DEFINITION OF THE MASS-TRANSFER MECHANISM IN THE EXTRACTION AND DISSOLUTION OF A SOLID PHASE

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A study has been made of the extraction of a solid substance from a spherical porous particle, in which there are two cases that differ in mechanism. In the first, the substance is extracted by physical dissolution of the solid phase in which case the region containing the soluble solid contracts continuously, while the part free from this substance steadily accumulates. In the second case, the extraction is based on chemical interaction with a reagent which diffuses through the boundary layer within the porous particle to the reaction surface.

Integral relationships for these processes have been used to derive a generalized kinetic equation from

$$\frac{t}{T} = \frac{1-3\varphi_0^2+2\varphi_0^3+\frac{6}{\varepsilon}(1-\varphi_0)+\frac{2}{\mathrm{Bi}}(1-\varphi_0^3)}{1+\frac{6}{\varepsilon}+\frac{2}{\mathrm{Bi}}}. \tag{1}$$

It has proved possible to identify (1) with the kinetic curves by linearization, which readily yields Bi and $\varepsilon$.

If we introduce the new function y and new argument x,

$$y = \frac{1-\frac{t}{T}\cdot\frac{1}{1-\varphi_0}}{\varphi_0(1-\varphi_0)}, \quad x = \frac{2\varphi_0-1}{1-\varphi_0} \tag{2}$$

then (1) becomes the linear equation

$$y = Ax + B, \tag{3}$$

where

$$A = \frac{1}{1+\frac{6}{\varepsilon}+\frac{2}{\mathrm{Bi}}}, \quad B = \frac{\frac{2}{\mathrm{Bi}}}{1+\frac{6}{\varepsilon}+\frac{2}{\mathrm{Bi}}}. \tag{4}$$

The essence of the method of establishing the mechanism consists in processing the initial kinetic equation $\varphi_0 = \varphi_0(t)$ for extraction from a monodisperse mixture of particles via (3), with determination of A and B, which then give the basic parameters Bi and $\varepsilon$ from

$$\mathrm{Bi} = \frac{2A}{B}, \quad \varepsilon = \frac{6A}{1-(A+B)}. \tag{5}$$

The following particular cases of (3) occur.

1. We have y = x for $\varepsilon = \infty$, Bi = $\infty$, A = 1, B = 0, which means that the following equation applies for the internal-diffusion mechanism:

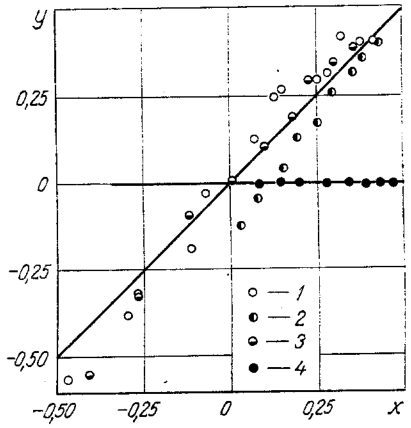

Fig. 1. Kinetic curves for extraction via various mechanisms: 1) alumina sinter with 15% NaOH, $d_{av}$ = 3.8 mm, t° = 60°C; 2) pyrite ore with kerosene, $d_{av}$ = 4 mm, t° = 110°C, 3) pyrite ore with tetrachloroethylene, $d_{av}$ = 4 mm, t° = 80°C; 4) sulfur with tetrachloroethylene, $d_{av}$ = 13 mm, t° = 60°C.

$$1-3\varphi_0^2+2\varphi_0^3=\frac{t}{T}. \quad (6)$$

2. We have $y \to 0$ for $\varepsilon \ll 1$, $A = \varepsilon/6 \ll 1$, $B = (\varepsilon/6)\cdot(2/\mathrm{Bi}) \ll 1$, which means that the chemical interaction between phases is described by

$$1-\varphi_0=\frac{t}{T}. \quad (7)$$

Equation (7) applies also for a dissolution in the diffusion region.

The experiments on Bi and $\varepsilon$ were performed with the following systems: pyrite ore with tetrachloroethylene, pyrite ore with kerosene, and alumina sinter with 15% NaOH solution; those on the dissolution kinetics were performed with spherical sulfur particles in tetrachloroethylene (Fig. 1).

The measurements were fitted to a relationship of the $y = f(x)$ form, which showed that all these processes involve mechanisms close to the internal diffusion type; on the other hand, the last case (pure dissolution) is described by the external-diffusion mechanism.

NOTATION

$\varphi_0$, dimensionless particle radius; t, current time; T, total extraction time; Bi, Biot number; $\varepsilon$, criterion defining the kinetics.

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# INTERNAL CHARACTERISTICS OF LOW-PRESSURE BOILING

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Experimental data are presented from high-speed photography of boiling in water and ethanol in large volumes at pressures of 0.0093-0.58 bar; the hot surface was provided by a Nichrome wire of diameter 0.5 mm. This wire was heated by an alternating current. The films show directly that there are considerable statistical differences between the cases. A method is given for calculating the mean values of the internal characteristics. The effects of the saturation pressure and specific heat flux on the mean internal characteristics indicate that the detachment diameter, growth time, and lag all increase as the pressure is reduced, while the concentration of nucleation centers, the nucleation frequency at such centers, the vapor formation rate, and the heat flux transported by the bubbles all fall. Similarly, the concentration of active centers, the nucleation frequency, the vapor formation rate, and the heat flux transported by the bubbles all increase with the heat flux, whereas the detachment diameter, growth time, and lag all fall.

An analysis is also presented for the growth and detachment conditions for single bubbles on this type of thin horizontal cylindrical heater. A characteristic feature of bubble growth is that a bubble reaches its maximum size before detachment, and this size is determined only by the excess enthalpy of the liquid thermal boundary layer present before the vapor bubble appears. The following formula has been derived for the maximum radius of the vapor bubble growing on such a heater:

$$R_{\max}=[0.5\delta_t\,(d_h+\delta_t)\,\mathrm{Ja}]^{0.5}.$$

This formula has been used in estimating the time spent by the bubble on the heater surface. The following relationship has been derived for the bubble growth:

$$R(\tau)=R_{\max}\left[1-\frac{8}{\pi^2}\exp\left(-\frac{\pi^2}{4}\mathrm{Fo}\right)\right].$$

This relationship is derived from a solution for the cooling of an unbounded thin-walled plate.

## NOTATION

$R_{max}$, maximum vapor-bubble radius; R, current vapor-bubble radius; $\tau$, time; $\delta_t$, thickness of the thermal boundary layer; $d_h$, heater diameter; Ja, Jacobs criterion; Fo, Fourier number.

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# CALCULATION OF TURBULENT VISCOSITY AND FLOW FOR A PLANAR CHANNEL

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The system of Reynolds equations may be closed by means of the transport equation for the total viscosity $K = (\nu + \nu_t)\nu^{-1}$ [1], where $\nu$ and $\nu_t$ are the molecular and turbulent viscosities. This equation has been applied to a pipe for the entire range in the transverse coordinate y. A system of equations is written for the flow of an incompressible liquid on the assumption that longitudinal diffusion and transverse pressure variation can be neglected.

The developed flow is then considered, with the momentum equation taking the form

$$du^+/dy_+ = (1 - y_+ Y_+^{-1}) K^{-1}, \tag{1}$$

and the equation for K becoming closed:

$$d^2K^2/dy_+^2 + 2A(1 - K^{-1})(1 - y_+ Y_+^{-1}) - 2B_1 K(K - 1)/y_+^2 f(y_+) = 0; \tag{2}$$

here $y_+ = yu^*/\nu$; $Y_+ = hu^*/\nu$; $u^+ = u/u^*$; $u^*$ is the dynamic velocity; and 2h is the channel width. Function $f = 1$ for $y_+ < 4$ and $y_+ > 26$ (for $4 \leqslant y_+ \leqslant 26$, see Table 1), while $A = 1.23$ and $B_1 = 12$ are universal empirical constants.

The boundary conditions for (2) are the following: $K(0) = 1$, $dK(Y_+)/dy_+ = 0$; when K has been determined, the distribution of the velocity and of the coefficient of friction $\lambda = 8(u_{av}^+)^{-2}$ can be derived from (1). Near the wall $(y_+ \leqslant 3)$, the following solution is derived $(x \equiv \sqrt{Ay_+})$:

$$K = 1 + \frac{35}{96}\left[\left(\frac{15}{x^2} - 6\right)\left(\frac{\sin x}{x} - \cos x\right) - 5\cos x\right] \simeq 1 + \frac{1}{190} y_+^4 - \cdots .$$

The solution far from the wall $(y_+ \geqslant 8\text{–}10)$ has been constructed subject to the condition $1 - K^{-1} \simeq 1$; (2) has also been solved numerically. The numerical calculations confirm the approximate analytical ones. The calculated K have been compared with the observed ones for tubes and channels. The agreement with experiment is good throughout the range $0 < y_+ < y_+$ provided that the mixing length L differs somewhat from the values predicted by a linear relationship in the region where the production and dissipation of turbulent energy are largest $(4 \leqslant y_+ \leqslant 26)$, the linear relation being $L = f^{1/2} y_+$; Table 1 gives values for $f(y_+)$:

TABLE 1

| $y_+$ | 4 | 6 | 10 | 14 | 18 | 22 | 26 |
|---|---|---|---|---|---|---|---|
| $f$ | 0,643 | 0,509 | 0,497 | 0,547 | 0,608 | 0,688 | 0,735 |

Good agreement with experiment is obtained with the distribution of $U^+$ for $0 < y_+ < 10^3$; the resistance coefficient $\lambda(\text{Re})$ agrees with experiment for an annular channel $(D_2/D_1 = 1.33)$ [2] but the values for tubes are somewhat higher than those found by Nikuradze.

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# REYNOLDS METHOD APPLIED TO DETERMINATION OF THE ONSET OF LAMINAR FLOW

A. A. Solov'ev UDC 532.542.4

In recent years, there has been considerable interest in reverse transition to laminar flow (laminarization) [1]. The paper discusses the scope for using an energy method, which differs from the common Orr—Kármán formulation in that Reynolds' original formulation is employed [2]. An example of a flow along the x direction is considered for a viscous incompressible liquid subject to a pressure and lying between parallel plates ($y = \pm b_0$); the mean motion is considered as plane-parallel, while the fluctuating motion is considered as planar. The liquid adheres to the walls, and the derivatives of the velocity with respect to the coordinates are zero there. The fluctuating motion is subject to the condition of periodicity in x ($\lambda$ is the period) and to the conditions that the fluctuating stresses are symmetrical. The working equation is

$$-\nu \int\limits_{-b_0}^{b_0} [l^4(\alpha^2+\beta^2)+2l^2(\dot{\alpha}^2+\dot{\beta}^2)+\ddot{\alpha}^2+\ddot{\beta}^2]\,dy = l \int\limits_{-b_0}^{b_0} \frac{d\bar{v}}{dy}(\alpha\dot{\beta}-\beta\dot{\alpha})\,dy, \tag{1}$$

where $\alpha$ and $\beta$ are the amplitudes of the pulsations, dots denoting derivatives with respect to y, $l = 2\pi/\lambda$ is the wave number of the pulsation, and $\nu = \eta/\rho$ is the viscosity. The derivative $d\bar{v}/dy$ is derived not from Poiseuille's law, but from the law $(u_{\max} - \bar{v})/v_* = 2.5 \ln b_0/y$. The right side of (1) is integrated by parts subject to the symmetry condition; we obtain

$$\frac{5v_* b_0}{\nu} = \frac{E_2}{E_1}, \tag{2}$$

where

$$\left.\begin{aligned} E_2 &= \int\limits_{-b_0}^{b_0} \{l^4(\alpha^2+\beta^2)+2l^2(\dot{\alpha}^2+\dot{\beta}^2)+\ddot{\alpha}^2+\ddot{\beta}^2\}\,ay \\ E_1 &= \frac{1}{2b_0}\int\limits_{-b_0}^{b_0}\frac{dy}{y^2}\int\limits_{-b_0}^{y} l(\beta\dot{\alpha}-\alpha\dot{\beta})\,dy \end{aligned}\right\}. \tag{3}$$

In accordance with the boundary conditions, one gets for the pulsation amplitudes that

$$\alpha = a\sin p + a\sin 3p, \qquad \beta = a\sin 2p + \frac{a}{2}\sin 4p,$$

where $p = \pi y/2b_0$; integration of (3) gives

$$E_1 = \frac{a^2\pi^2 L}{16b_0^3}\left\{3\,\mathrm{Si}\left(\frac{\pi}{2}\right)+3\,\mathrm{Si}\left(\frac{3}{2}\pi\right)-\mathrm{Si}\left(\frac{5}{2}\pi\right)-\mathrm{Si}\left(\frac{7}{2}\pi\right)\right\},$$

$$E_2 = \frac{a^2}{b_0^3}\left(\frac{\pi}{2}\right)^4\{3{,}25L^4+36L^2+162\},$$

where $L = (2b_0/\pi)l$.

After transferring by standard means from the dynamic velocity $v_*$ to the mean speed u one gets

$$Re = \frac{2b_0 u}{\nu} = \frac{37.4\{L^4+11L^2+50\}}{L}.$$

The minimum is defined by $L_{\min} = 1.62$, which gives $Re_{\min} = 2000$; similar calculations can be performed for other types of flow.

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# MEASUREMENT OF HEAT-TRANSFER COEFFICIENTS FOR STEAM BUBBLES RISING IN SUPERHEATED WATER

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Little has been published on heat-transfer coefficients for steam bubbles growing in water; the usual value employed is $\bar{\bar{\alpha}} = 18.5 \cdot 10^3$ W/(m$^2$·deg), which was derived in [1] as the means of values derived for six bubbles of volumes up to 200 mm$^3$.

A special device [2, 3] has been used to measure $\alpha$ for 15 large bubbles (volumes up to 50 cm$^3$) rising in boiling water at $P_0 = 1$ bar. Motion-picture photography and temperature measurement have been used to calculate the instantaneous heat-transfer coefficient $\alpha_j(\tau)$, where $j$ is the number of the bubble and $\tau$ is time. Averaging with respect to $\tau$ then gives the mean values $\bar{\alpha}_j$, and then averaging with respect to $j$ gives the mean result for all bubbles $\bar{\bar{\alpha}} \equiv (\bar{\alpha}_j) = 57 \cdot 10^3$ W/(m$^2$·deg). The data were processed by computer on the basis of spectral analysis and smoothing.

The data indicate that the different modes of flow cause the values for $\bar{\alpha}_j$ to differ considerably, as do the values of $\alpha$ for a given bubble at different $\tau$. Similar observations have been made previously [1]. It is found that the result $\bar{\bar{\alpha}} = 57 \cdot 10^3$ as the standard deviation $\Delta(\bar{\bar{\alpha}}) = 39 \cdot 10^3$, i.e., the two are of the same order of magnitude, and this standard deviation is due not to instrumental errors, but to differences in the $\alpha_j(\tau)$ curves.

This indicates that it is insufficient to represent the heat-transfer data for bubbles growing in boiling water simply by means of an average value $\bar{\bar{\alpha}}$, since bubbles must be differentiated into classes in accordance with the growth parameters.

This means that the result $\bar{\bar{\alpha}} = 57 \cdot 10^3$ should not be considered as replacing the result $\bar{\bar{\alpha}} = 18.5 \cdot 10^3$ from [1], but merely as supplementing it (on the basis of the above standard deviation) for the case of large bubbles growing in a rapidly boiling liquid.

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# DETERMINATION OF THE THERMOPHYSICAL CHARACTERISTICS OF SUBSTANCES BY THE EFFECTIVE-PARAMETER METHOD IN TEMPERATURE-TIME INTERVAL TECHNIQUES

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UDC 536.2.083

The effective-parameter method, which allows one to calculate thermophysical characteristics from working tables employed in primary methods, while retaining conditions appropriate to those of the experiments used in the temperature–time interval technique, is described. The effective-parameter method allows one to determine thermophysical characteristics of corrosive substances, resins, electrolytes, volatile liquids, and solid insulators, as well as of rocks and materials of high thermal conductivity, on specimens of small thickness. The method is based on analysis of the restricted applicability of additivity for thermal resistances under nonstationary conditions.

The theory of the method includes a correction for the contact thermal resistance.

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# PARTICLE-SIZE DISPERSION PREDICTION FOR SOLID PARTICLES OF CONDENSATION ORIGIN

A. G. Sutugin and É. I. Kottsev

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The paper presents a method of the calculating particle-size dispersion for the case where the specific surface of the deposited product is determined by the capacity of the particles to fuse on deposition. It is assumed that the particles are formed together as the temperature is reduced. The particle sizes increase with time, while the falling temperature means that the particles gradually lose the capacity to fuse together, so the mode of coagulation tends to alter, since particle collision ultimately results only in clumps held together by adhesive forces, with no corresponding change in particle size. The temperature corresponding to this point can be derived by measuring the stability of the powder under heating. Such measurements give the maximum specific surface that can persist at a given temperature, $S_{max}(T)$.

Numerical solution of the equations for condensation and coagulation have been used in determining the behavior of the particle size and the specific surface as a function of time:

$$S_k(\theta) = A\theta^{-2/5}(36\pi/\rho^2 m), \tag{1}$$

where A is the coefficient dependent on the Gamaker constant, θ is dimensionless time, ρ is density, and m is molecular mass.

The relationship between θ and the physical time is given for coagulation in a turbulent jet by

$$\theta=[2.66\ d_0/u_0]^{-1}K(1,\ 1)\,x_1^0\omega, \tag{2}$$

where w is the dilution of the hot mixture by cooling gas, $x_1^0$ is the initial concentration of the vapor molecules, $u_0$ is the flow speed, $d_0$ is the nozzle diameter, and K(1, 1) is a coefficient for the collisional frequency of the vapor molecules. Since w and T are known as functions of t, one can construct curves for $S_{max}(\theta)$ and $S_k(\theta)$, whose intersection defines the desired specific surface. The method can be applied to existing data on the condensation of silver and molybdenum trioxide vapors. In the case of $MoO_3$, the method systematically predicts S too large by factors of 2-3. As regards silver, it was found that the temperature cannot be reduced sufficiently rapidly for coagulation to be inhibited, i.e., the

$S_{max}(\theta)$ and $S_k(\theta)$ curves do not intersect. This illustrates the scope for using the method to determine the necessary equipment size.

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# MEASUREMENT OF ELECTRICAL POTENTIALS IN ELECTROHYDRODYNAMIC FLOWS

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UDC 534.22:536.24.01

Recent researches have placed much emphasis on the interaction between hydrodynamic and acoustic motions [1], and therefore measurements have been made on the Debye effect. A description is given of the experiments and of the method of measuring the electrical potentials arising in solutions of electrolytes in the presence of ultrasonic waves; the potentials were measured in standing waves. Parasitic interference was provided by a high-frequency oscillator. The probe signal was amplitude-modulated by the signal due to the Debye effect. The modulation amplitude $\Delta\phi = \phi_{max} - \phi_{min}$ was measured, and the ratio of this quantity to the amplitude unit W for the particle vibration velocity was calculated. The latter was derived from the measured generator voltage. The ultrasound frequency was 1.7 MHz. The probes gave no signal in nonpolar liquid. The potentials given by three aqueous solutions (KCl, NaCl, and $MgSO_4$) are shown in Fig. 1. Only KCl solution satisfied Debye's theory, which indicates that the effect should be independent of the concentration at low concentrations. The fall in the apparent mass difference between the anions and cations at high concentrations should reduce the effect which appears to occur for NaCl solution. The behavior of $MgSO_4$ solution may be explained in terms of a diffusion correction, which makes an increasing contribution on account of the difference in ionic mobilities. On the whole, the results indicate that there are deviations from Debye's theory at high concentrations.

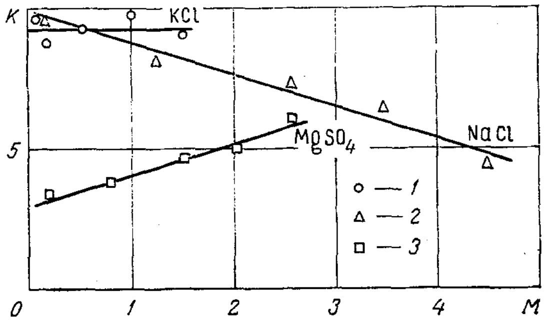

Fig. 1. Amplitude of the electrical potential $K = \Delta\varphi/w$ in μV/cm/sec as a function of salt concentration M for aqueous electrolyte solutions: 1) KCl; 2) NaCl; 3) $MgSO_4$.

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# ANALYSIS OF INHOMOGENEOUS SEMICONDUCTOR STRUCTURES BY METHODS OF NONEQUILIBRIUM THERMODYNAMICS

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UDC 621.382

An attempt is made to show that known methods of the thermodynamics of irreversible processes (the Onsager theory) may be used, under certain assumptions, for the analysis of complex semiconductor equipment with a certain number of p—n junctions. On the basis of the Second Law of Thermodynamics and the assumption that the semiconductor consists of a set of fixed lattice points and mobile holes and electrons, an expression is obtained for the rate of growth of entropy:

$$T\theta = -\frac{\vec{J}'_q}{T}\operatorname{grad} T + \frac{1}{q}\vec{J}_{n0}\operatorname{grad}\left(kT\ln\frac{n_0}{N_e}\right) - \frac{1}{q}\vec{J}_{p0}\operatorname{grad}\left(kT\ln\frac{P_0}{N_v}\right) +$$

$$+\frac{1}{q}\sum_{v=1}^{w}\vec{J}_{nv}\operatorname{grad}\left(kT\ln\frac{n_v}{N_c}\right) - \frac{1}{q}\sum_{v=1}^{w}\vec{J}_{pv}\operatorname{grad}\left(kT\ln\frac{P_v}{N_v}\right) -$$

$$-\vec{J}_0\operatorname{grad}\varphi_0 - \sum_{e=1}^{m-1}\vec{J}_e\operatorname{grad}\varphi_e - \sum_{v=0}^{w}\left(kT - kT\ln\frac{n_v}{N_c} + q\varphi\right)\gamma_{nv} - \sum_{v=0}^{w}\left(kT + kT\ln\frac{P_v}{N_v} - q\varphi\right)\gamma_{pv}.$$

Here $\vec{J}'_q$ is the generalized heat flow; $\vec{J}_{no}$, $\vec{J}_{po}$, $n_o$, and $p_o$ are the current densities and hole and electron concentrations caused by the fields of space charges in the structure (background values); $\vec{J}_1$ and $\varphi_0$ are the total density of the background current and electrical potential at each point of space; and $\vec{J}_{nv}$, $\vec{J}_{pv}$, $n_v$, $P_v$, $\vec{J}_e$, and $\varphi_e$ are values of the current density, charge-carrier concentration, and electrical potential caused by the corresponding p—n junction (v) and electrode (e). It is assumed that the structure contains w n—p junctions and m electrodes, one of which (numbered below m) has a fixed potential (is grounded). The rest of the notation is conventional.

In this representation, the number of vector flows and thermodynamic forces for the transfer equation can be chosen to agree with the number of electrodes and p—n junctions. The kinetic coefficients are determined on the assumption that the electron and hole mobilities are matrix values in the field of these forces.

It is shown that the method outlined can be used to derive generalized Ebers—Moll equations for structures of any complexity, taking into account the bulk resistance of the region and leakage at p—n junctions. Examples of the formulation of such equations are given.

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# SLOW NONISOTHERMAL MOTION OF A NONLINEARLY VISCOPLASTIC MEDIUM IN A CIRCULAR CYLINDRICAL TUBE

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The effect of the heat of internal friction on the hydraulic characteristic of laminar motion of a nonlinearly viscoplastic medium in a circular tube, taking into account the change in plastic viscosity and limiting shear stress with temperature, is considered.

The solution reduces to a system of nonlinear differential equations:

$$\frac{1}{\rho}\cdot\frac{d}{d\rho}\left(\rho\,\frac{d\theta_1}{d\rho}\right)=0,\quad 0\leqslant\rho\leqslant\rho_0, \tag{1}$$

$$\frac{1}{\rho}\cdot\frac{d}{d\rho}\left(\rho\,\frac{d\theta_2}{d\rho}\right)+N\rho\,[(\rho\theta_2)^{\frac{1}{n}}-\rho_0^{\frac{1}{n}}]^n=0,\quad \rho_0\leqslant\rho\leqslant 1, \tag{2}$$

$$-\frac{dU_z}{d\rho}=\frac{\Delta PR^2}{2l\mu_w}\,[(\rho\theta_2)^{\frac{1}{n}}-\rho_0^{\frac{1}{n}}]^n \tag{3}$$

with corresponding boundary conditions. Here $N = (\Delta PR^2/2l)^2/\lambda T_w\mu_w$; $\theta_i$ is the dimensionless temperature; $\Delta P/l$ is the pressure drop; R is the tube radius; $\rho$ is the dimensionless radius; $\rho_0$ is the dimensionless core radius; n is the nonlinearity parameter; $\mu_w$ is the plastic viscosity at the wall temperature $T_w$; $U_z$ is the longitudinal velocity; and $\lambda$ is the thermal conductivity.

The system (1), (2) is solved using the comparison theorem, which involves finding upper and lower bounds for the differential equations, between which the true solution must lie. Rough calculations show that the relative error between the upper and lower solutions depends on the parameters N, $\rho_0$, and n; increase in N leads to increase in the error and increase in $\rho_0$ and n, to reduction in the error. However, in the present investigation the arithmetic mean of the upper and lower solutions is used, since this solution differs from the accurate solution for a particular case (a Newtonian medium) by not more than 3%.

Calculations show that increase in the limiting shear stress leads to reduction in the effect of the temperature dependence of the viscosity and limiting shear stress on the temperature, flow core, and flow rate. With increase in the nonlinearity parameter n, the rate of reduction in these values sharply increases.

Comparison of the results with those obtained for the analogous problem with constant rheological parameters shows that in the case of temperature-dependent rheological coefficients there is a marked increase both in the temperature of the viscoplastic medium in the region of the quasisolid core and in the flow rate; with rise in n, the difference between the two sets of results decreases.

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# SOLUTION OF THERMAL-BALANCE EQUATIONS FOR MULTIPASS HEAT EXCHANGERS

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An analytic solution is obtained for the system of thermal balance equations for a multipass heat exchanger with one path in the body and an arbitrary number of tube passes

$$\frac{dT}{dx}=\frac{a}{N}\left(\sum_{i=1}^{N}\theta_i-NT\right),$$

$$\frac{d\theta_i}{dx}=(-1)^i\frac{b}{N}(T-\theta_i),\ i=1,\ \ldots,\ N.$$

$$T(0)=T_s;\ \theta_1(1)=\theta_s,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\theta_{2j}(0)=\theta_{2j-1}(0);\ \theta_{2j+1}(1)=\theta_{2j}(1),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$0<j<\frac{N}{2}.$$

Here N is the number of tube passes; T and $\theta_i$ are the heat-carrier temperatures in the intertube space and in the i-th tube pass; $a$ and b are dimensionless coefficients; and the subscripts s and f denote the starting and final values of the temperature.

An explicit expression is given for the temperature efficiency factor (TEF) of the heat exchanger: $(T_f-T_s)/(\theta_s-T_s)=\Phi(a,\ b,\ N)$ for odd and even N; check calculations for multipass heat exchangers are considerably simpler using this equation than by existing iterative methods. According to a note by R. Bauman,

$$\lim_{N\to\infty}\Phi(a,\ b,\ N)=\Phi(a,\ b),$$

where $\Phi(a,\ b)$ is the TEF of a heat exchanger with crossover flow for total mixing of each heat carrier in any transverse current line of the section. When N is even, $\Phi(a,\ b,\ N)$ is close to $\Phi(a,\ b)$ for all $a$ and b at fairly low values of N. For odd N, the convergence is not uniform with respect to $a$ and b and, even at high values of N, there are values of the parameters at which the use of $\Phi(a,\ b)$ instead of $\Phi(a,\ b,\ N)$ to simplify the calculation is impossible (Fig. 1).

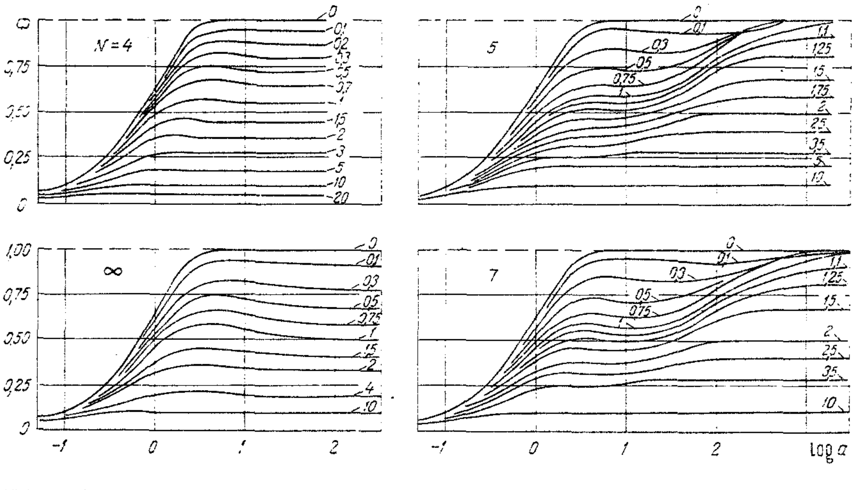

Fig. 1. TEF of heat exchangers with one pass in body and N tube passes. The dependence of $\Phi=\Phi(a,\ xa,\ N)$ on $a$ is shown for various balues of $x=b/a$ (shown above the curves).

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# NUMERICAL SOLUTION OF NONLINEAR BOUNDARY-VALUE PROBLEMS OF THERMAL AND ELECTRICAL TRANSFER IN MULTILAYER SYSTEMS

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UDC 517.9+536.2+621.362

The transfer and interconversion of heat and electricity in multilayer two-blade systems is described mathematically by a nonlinear system of one-dimensional differential equations of parabolic type:

$$\left.\begin{array}{c} c_{s,v}\gamma_{s,v}\dfrac{\partial T_{s,v}}{\partial t}=\dfrac{\partial}{\partial x}\left(k_{s,v}\dfrac{\partial T_{s,v}}{\partial x}\right)-\tau^J_{s,v}J_{s,v}\dfrac{\partial T_{s,v}}{\partial x}+\rho_{s,v}J^2_{s,v}+q_{s,v}, \\ X_{s-1}<x<X_s;\ s=1,\ 2,\ \ldots,\ M;\ X_0=0;\ X_M=1;\ 0<t<\infty;\ v=1,\ 2, \end{array}\right\} \tag{1}$$

with initial and boundary conditions (at the ends of the system)

$$\left.\begin{array}{c} T_{s,v}(x,\ 0)=\Phi_{s,v}(x); \\ T_{1v}(0,\ t)=\psi_v(t),\ T_{M,v}(1,\ t)=\varphi_v(t) \end{array}\right\} \tag{2}$$

and matching conditions at points of contact of different layers ($x = X_s$, $s = 1, 2, \ldots, M - 1$; $v = 1, 2$)

$$\left.\begin{array}{c} T_{s,v}(X_s,\ t)=T_{s+1,v}(X_s,\ t); \\ \left(-k_{s,v}\dfrac{\partial T_{s,v}}{\partial x}+\alpha^J_{s,v}J_{s,v}T_{s,v}\right)\Bigg|_{X_s}=\left(-k_{s+1,v}\dfrac{\partial T_{s+1,v}}{\partial x}+\alpha^J_{s+1,v}J_{s+1,v}T_{s+1,v}\right)\Bigg|_{X_s} \end{array}\right\}. \tag{3}$$

The subscripts s and v give the number of the layer and the blade, respectively; $X_s$ is the coordinate at the edge of the layer s; $T_{s,v}$ is the temperature; $k_{s,v}L^2 = \lambda_{s,v}$ is the thermal conductivity; $L\tau^J_{s,v}$ is the Thomson coefficient; $L\alpha^J_{s,v}$ the Seebeck coefficient; the specific heat $c_{s,v}$, electrical resistivity $\rho_{s,v}$, and density $\gamma_{s,v}$ are functions of the temperature, and are often represented in the form of polynomials in $T_{s,v}$; $\rho_{s,v}J^2_{s,v}$ is the change in Joule heat liberated in the layer when an electric current $I_{s,v} = |J_{s,v}|S_{s,v}$ flows in it ($S_{s,v}$ is the cross-sectional area of the layer); $\alpha^J_{s,v}J_{s,v}T_{s,v}$ is the Peltier heat; $q_{s,v}$ is an external heat source (sink); and L is the length of each of the two blades.

The electrical current density $J_{s,v}$ is defined in terms of $T_{s,v}$ using the relation (the layers in each blade are both electrically and thermally in series; the two blades are electrically in series but thermally in parallel)

$$I\equiv I_{s,v}=\sum_{s,v}E_{s,v}/\left(R_L+\sum_{s,v}R_{s,v}\right), \tag{4}$$

where the thermo-emf appearing in the system because of the Seebeck effect is

$$E_s=E_{s,1}+E_{s,2}=L\left[(n_E)_{s,1}\int\limits_{T_{X_{s,1}}}^{T_{X_{s-1,1}}}\alpha^J_{s,1}(T)\,dT-(n_E)_{s,2}\int\limits_{T_{X_{s,2}}}^{T_{X_{s-1,2}}}\alpha^J_{s,2}(T)\,dT\right] \tag{5}$$

and the internal electrical resistance is

$$R_s=R_{s,1}+R_{s,2}=L\left[\frac{(n_E)_{s,1}}{S_{s,1}}\int\limits_{X_{s-1}}^{X_s}\rho_{s,1}(T)\,dx+\frac{(n_E)_{s,2}}{S_{s,2}}\int\limits_{X_{s-1}}^{X_s}\rho_{s,2}(T)\,dx\right]; \tag{6}$$

$R_L$ is the external load in the circuit; $(n_E)_{s,v}$ is the number of identical elementary transformers in the layer which are electrically in series and thermally in parallel.

The coefficients c, γ, k, $\tau^J$, ρ, and $\alpha^J$ have discontinuities of the first kind at the edges of the layers; for $\alpha^J \neq 0$ [see Eq. (3)] the conductive heat flow ($-\lambda\partial T/\partial x$) is also discontinuous at these points. The solution is sought in the class of piecewise-smooth functions.

For the first boundary-value problem (1)-(3) a homogeneous, conservative, and monotonic purely implicit difference scheme of continuous calculation is constructed.

The results of machine calculation on an M-222 computer confirm that the difference scheme developed is absolutely stable, converges sufficiently rapidly, guarantees an accuracy

$O(h^2 + \tau)$, and allows the investigation of both steady and unsteady operation of the appropriate equipment (electrical generators, refrigeration and heating pumps, etc.).

The calculation scheme is described (the program — in Ta-28 input language — is included in Gosfond, the State Bank of Algorithms and Programs) and results are given for a specific seven-layer system of two blades. In the special one-layer case, this system reduces to the known example and reproduces a number of literature data.

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# SOLUTION OF STEADY HEAT-CONDUCTION PROBLEMS FOR SEMIPLANES BY PAIRED INTEGRAL EQUATIONS

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UDC 517.946

In [1, 2] paired integral equations were used to solve problems of diffraction theory. Additional use of the factorization method allows the corresponding problems of heat-conduction theory to be considered. The problem reduces to solving the paired integral equations

$$\begin{aligned}&\int_0^\infty M(\nu)\cos \nu x\,d\nu = 1,\quad 0 < x < a,\\ &\int_0^\infty M(\nu)(\nu + h)\cos \nu x\,d\nu = 0,\quad a < x < \infty,\end{aligned} \tag{1}$$

where $M(\nu)$ is a real function; $h$ and $a$ are positive constants.

By factorization of the piecewise-holomorphic function

$$G(\nu) = \begin{cases} h + \nu, & \operatorname{Re}\nu \geqslant 0,\\ h - \nu, & \operatorname{Re}\nu \leqslant 0\end{cases} \tag{2}$$

in the form [3]

$$G(\nu) = G_+(\nu)\,G_-(\nu)$$

it is possible to obtain a Fredholm integral equation of the second kind for the auxiliary function $\varphi(t)$:

$$\varphi(t) = \int_a^\infty K(t + \tau)\,d\tau + \int_a^\infty \varphi(\tau)K(t + \tau)\,d\tau, \tag{3}$$

$$a \leqslant t < \infty,$$

where

$$K(t) = \frac{-1}{2\pi}\int_{-\infty - i\varepsilon}^{+\infty - i\varepsilon} \frac{G_+(\nu)}{G_-(\nu)} \exp(-i\nu t)\,d\nu,$$

$$t > 0,\ \varepsilon > 0.$$

The use of Eq. (3) is convenient at large values of the heat-transfer parameter. The case of small values was considered in [4, 5].

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# ITERATIVE METHOD OF OPTIMAL LINEARIZATION

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As is known (see [1, 4], for example), existing approximate analytic methods of solving nonlinear heat-conduction problems have no great advantage over methods of linearization.

The general idea of linearization is to replace the initial nonlinear problem by a linear problem in such a way that the solution of the linear problem is in some sense close to the solution of the nonlinear problem. The choice of a specific linear problem to provide a good approximation is largely subjective.

The paper proposes an iterative method of linearization of nonlinear heat-conduction problems, which generalizes and refines the method of optimal linearization [2]. The principle underlying the method is illustrated by considering problems with nonlinearity of the second kind (according to the classification of [3]). These problems arise in determining the temperature field in solids in the presence of radiant, convective, or combined heat transfer with a medium. By linearizing the initial problem, a formula is obtained to determine the constant value of the heat-transfer coefficient.

A program realizing the iterative method of optimal linearization is developed for the MIR-2 computer.

The method is used to calculate the heating of steel components in molten $NaNO_3$. The solution obtained is no less accurate than that obtained by the small-parameter method [3], and more accurate than that given by the method of optimal linearization.

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